NON-ANALYTIC FUNCTIONS OF A COMPLEX VARIABLE REPRESENTABLE BY LEBESQUE-STIELTJES INTEGRALS WITH A CAUCHY KERNEL

BY

JOE W. WRAY
B.S., University of Georgia, 1939
M.S., University of Georgia, 1944
M.A., University of Illinois, 1948

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SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
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NON-ANALYTIC FUNCTIONS OF A COMPLEX VARIABLE REPRESENTABLE BY LEBESQUE-STIELTJES INTEGRALS WITH A CAUCHY KERNEL

In section II of this paper a non-regular set of positive measure has been constructed which has only frontier points. Using methods similar to those used by Trjitzinsky a non-analytic function, which is the limit of a sequence of analytic functions, has been represented over this set by an iterated Lebesque-Stieltjes integral with a Cauchy kernel. $f(\alpha)=\iint \frac{d\mu}{z-\alpha}$ Here μ is an absolutely continuous set function and its explicit form is

$$\mu(X)=\iint_X g(x,y)\,dxdy.$$

In section three a comparison is made of differentiability conditions of series of the type $\sum_{\nu=1}^{\infty} \frac{b_\nu}{\alpha_\nu-\alpha}$. Here the Cauchy kernel representation is $\sum_{\nu=1}^{\infty} \frac{b_\nu}{\alpha_\nu-\alpha} = \iint \frac{d\mu}{z-\alpha}$. μ is considered both as a singular and mixed function of sets. In the last part of this section it is further shown that $\iint \frac{d\mu}{z-\alpha}$ is "differentiable" if $\iint \frac{d\mu}{(z-\alpha)^2}$ exists. Here quotation means that the path of approach to α is restricted to a sequence of points taken from a curve having a unique tangent at α.

The last section begins with Wolff's work

$$\frac{1}{\alpha_\nu-\alpha} = \frac{1}{\pi r_\nu^2}\iint_{S_\nu}\frac{dxdy}{z-\alpha}$$

and develops an integral representation of the series $\sum_{\nu=1}^{\infty} \frac{b_\nu}{(\alpha_\nu-\alpha)^m}$. That representation

$\sum_{\nu=1}^{\infty} \frac{b_\nu}{(\alpha_\nu - \alpha)^m} = \iint \frac{d\mu}{(z - \alpha)^m}$ is possible if the series

$\sum_{\nu=1}^{\infty} \nu^{m-1} \overline{S}\, S(\alpha, \frac{r_0}{\nu})|b_\nu|$ converges. The notation $S(\alpha, \frac{r_0}{\nu})|b_\nu|$ represents the sum of the absolute values of all the b_ν which correspond in a one-to-one way to the α_ν in the set $S(\alpha, \frac{r_0}{\nu})$. Here again μ is considered both as a singular and a mixed function of sets.

VITA

Joe Willie Wray was born on October 27, 1912, in Polk County, Georgia near Cedartown. Although he graduated from Cedartown High School in 1929 he did not enter Middle Georgia College, a junior college in the University of Georgia System, until 1934. The two years immediately following graduation from junior college in 1936 were spent in teaching in a consolidated school in his home county. Having studied at University of Georgia during the summer of 1937 and the entire year following September 1938, he received the degree Bachelor of Science in August, 1939. The three following years were spent in high school teaching in Georgia and South Carolina. The summers of 1941, 1942, and 1943 were devoted to graduate study at University of Georgia, the degree Master of Science being received in August, 1944. He was instructor in mathematics at North Carolina State College during the years 1942-1944 and again during the year 1946-1947. He was a member of the faculty at Flora Macdonald College during the academic years 1944-1946. Graduate work was carried on at University of North Carolina during the summers of 1944, 1945, and 1946. Graduate study was begun at University of Illinois in June, 1947. He held a half-time graduate assistantship for five semesters and one summer, terminating in August, 1950. The degree Master of Arts was received in October, 1948. The second semester of the academic year 1948-1949 was spent in New York State where he was Acting Assistant Professor of Mathematics at State Teachers College of State University of New York. Since September, 1950 he has been Assistant Professor of Mathematics at University of Idaho.